

Topological Stability of Stored Optical Vortices

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We report an experiment in which an optical vortex is stored in a vapor of Rb atoms. Due to its 2π phase twist, this mode, also known as the Laguerre-Gauss mode, is topologically stable and cannot unwind even under conditions of strong diffusion. For comparison we stored a Gaussian beam with a dark center and a uniform phase. Contrary to the optical vortex, which stays stable for over $100\mu s$, the dark center in the retrieved flat-phased image was filled with light after storage time as short as $10\mu s$. The experiment proves that higher electromagnetic modes can be converted into atomic coherences, and that modes with phase singularities are robust to decoherence effects such as diffusion. This opens the possibility to more elaborate schemes for two dimensional information storage in atomic vapors.

Topological defects are abundant in diverse coherent media, ranging from neutron stars [1, 2], superconductors [3], superfluid ^4He and ^3He [3, 4], Bose-Einstein condensates [5], effective two dimensional Rb atoms [6] and light [7]. A topological defect is a spatial configuration of a vector field that defines a distinct homotopy class [8], it is therefore stable against continuous deformations which cannot cause it to decay or to “unwind”. It has been suggested that this robustness can be harnessed to combat decoherence effects by using topological defects as qubits in a quantum computation scheme [9, 10].

Perhaps the simplest example of a topological defect is a vortex. A vortex in a coherent media occurs whenever there is a 2π dislocation of the phase around a given point in space [4, 5, 8]. In particular, a light vortex is an electromagnetic mode with a phase $e^{i\theta(x,y)}$, where $\theta(x,y) = \arctan(y/x)$, is defined in the plane perpendicular to the propagation direction \hat{z} . There exist a family of free space solutions of Maxwell’s equations, known as the Laguerre-Gauss modes, that have this phase factor [7, 11].

In this work we report an experiment in which a light vortex was stored in “hot” Rb vapor and then retrieved. We show that due to its topological stability, the retrieved light beam maintains its phase singularity, thus staying dark at the center in a regime of strong diffusion for storage times up to $110\mu s$. For comparison, we stored a Gaussian beam with a uniform phase and a dark center, and observed that the center was filled with light after storage of only $10\mu s$ due to diffusion of atoms to the center. We suggest to use this topological robustness for improved storage of two dimensional information e.g. images. The prospect of creating more elaborate defects is also briefly discussed.

When storing light in an electromagnetically induced transparency (EIT) medium, the information contained in a weak “probe” field, namely its amplitude and phase, is continuously transformed onto the atomic coherence [12, 13]. The propagation of the combined light-matter excitation in the medium (“dark-state polariton”), its deceleration down to zero velocity for a controlled period of

time (storage), and the subsequent retrieval of the probe pulse, are controlled by switching a strong “pump” field. In the simplest case, this coherent atomic medium is described as a three-level Λ -system with the probe and pump fields resonant with the two optical transitions $1, 2 \leftrightarrow 3$ (see Fig. 1.a). In a density matrix formalism the dark-state is identified with the ground level coherence $\rho_{12}(\mathbf{r}, t) = (-g/\Omega) E(\mathbf{r}, t)$, where g is the atom-field coupling constant, Ω is the Rabi frequency of the pump and $E(\mathbf{r}, t)$ is the slowly varying amplitude of the probe field [12]. Assuming the pump field has a constant phase relative to the probe, we note that the atomic coherence follows the complex probe field $E(\mathbf{r}, t)$, and hence for a general transverse electromagnetic mode it may be different for each point on the transverse plane.

A particularly interesting electromagnetic mode is the so called Laguerre-Gauss (LG) mode, also known as an optical vortex or helical beam. The electric field of the LG_m^n mode is given in polar coordinates (in the $z = 0$ plane) by [14] $E_m^{\text{helical}}(r, \theta) = A_m(r, w_0) e^{-im\theta}$ where w_0 is the waist, m is the winding number and $A_m(r, w_0) = \frac{1}{w_0} \sqrt{\frac{2P}{\pi m!}} (\sqrt{2}r/w_0)^m \exp(-r^2/w_0^2)$ is the “ring shaped” radial cross-section with P the total intensity. The LG modes have some unique properties, e.g. carrying orbital angular momentum [7, 11]. The aspect relevant for this work is that this mode has a $2\pi m$ phase twist around its dark center. The dark center of the light vortex — a result of the phase singularity — can be intuitively understood for the $m = 1$ case, as a destructive interference between any two diametrically opposed (antipodal) pair of points around the center, since any such pair has a π phase shift. The winding number m is a topological invariant characterizing this $U(1)$ defect, which cannot change under continuous deformations of the probe field. This suggests that by storing a light vortex in the atomic coherences, we can create a coherence field which is robust to diffusion, since diffusion is, on average, a continuous process.

To account for the effect of diffusion on the retrieved signal, we assume that the internal state of each individ-

ual atom does not change as a result of diffusion, and so it carries with it the stored complex amplitude. The ensemble average of the different atoms arriving to the same small macroscopic volume will determine the retrieved probe field [15]. Therefore the atomic diffusive motion for a large number of atoms can be described as a continuous diffusion process of the density matrix field, by adding the term $\dot{\rho}_{\text{diff}} = D\nabla^2\rho$ to the Bloch equations, where D is the diffusion coefficient. Eventually, the coherence field after a storage time t , and neglecting all other decay mechanisms, is calculated by propagating the initial coherence forward in time, using the three-dimensional diffusion propagator, $G(\mathbf{r}, \mathbf{r}', t) = (2\pi Dt)^{-3/2} \exp[-(\mathbf{r} - \mathbf{r}')^2 / (4Dt)]$. The retrieved probe field, E_{ret} , is given by $E_{\text{ret}}(\mathbf{r}, t) = (-\Omega/g)\rho_{12}(\mathbf{r}, t)$.

Taking the initially stored coherence field to be $\rho_{12}(r, \theta, t=0) = (-g/\Omega) E_m^{\text{helical}}(r, \theta)$, the expected field after diffusion of duration t is a scaled LG mode,

$$\rho_{12}^{\text{helical}}(r, \theta, t) = \frac{(-g/\Omega)}{\sqrt{s(t)^{m+1}}} A_m\left(r, \sqrt{s(t)}w_0\right) e^{-im\theta}, \quad (1)$$

where $s(t) = (w_0^2 + 4Dt)/w_0^2$ is a scaling factor. Note that at the center $\rho_{12}(r=0, t) = 0$ for all t . For the sake of comparison, we examine the diffusion of a Gaussian beam with a dark hole of radius r_0 at the center, and a “flat” (constant) phase, $E^{\text{flat}}(r) = \Theta(r - r_0) A_0(r, w_0)$, where $\Theta(r)$ is the step function. The field of atomic coherence after time t for this beam profile is given by

$$\rho_{12}^{\text{flat}}(r, t) = \frac{(-g/\Omega)}{2Dt} \sqrt{\frac{2P}{\pi w_0^2}} \int_{r_0}^{\infty} dr' r' e^{-\frac{r^2 + sr'^2}{4Dt}} I_0\left(\frac{rr'}{2Dt}\right), \quad (2)$$

where $I_0(x)$ is the modified Bessel function of order zero [16]. Taking $r_0 = w_0/2$ the coherence at the center ($r=0$) is proportional to $\exp[1/(1-s)/4]/s$, so the hole is expected to be filled at $t \approx 0.15w_0^2/D$.

In order to store the light in this experiment, we employed the EIT within the $D1$ transition of ^{87}Rb [17]. The energy level scheme is presented in Fig. 1.a, showing the Λ -system and the pump and probe transitions. The experimental apparatus is depicted in Fig. 1.b. An external-cavity diode-laser (ECDL) was locked to the $F = 2 \rightarrow F' = 1$ transition. The laser was divided into two beams of perpendicular linear polarizations, the pump and the probe, using a polarizing beam-splitter (PBS). The pump beam was passed through an acousto-optic modulator (AOM) and the first order was used for the experiment, allowing us to control both the frequency and the intensity of the pump. The pump was shaped as a Gaussian beam with a waist of $w_{\text{pump}} = 2.2\text{mm}$ (at the center of the vapor cell) and a total intensity of 1.1mW . The probe channel included a second AOM in a similar arrangement, as well as an off-axis holographic binary mask, used to shape the probe as a helical beam with $m = 1$. The waist of the helical probe was set to

$w_0 = 670\text{ }\mu\text{m}$ at the center of the vapor cell and its intensity was $32\text{ }\mu\text{W}$. The small size of the probe insured that it will experience a nearly constant pump intensity and phase, and also guaranteed the dominance of the diffusion mechanism over other decoherence mechanisms. The pump and the probe beams were recombined on a second PBS, and co-propagated toward the vapor cell. A quarter wave-plate before the cell converted the pump and the probe from linear to σ^+, σ^- polarizations, suitable to the transitions presented in Fig. 1.a. A 5 cm long vapor cell containing isotopically pure ^{87}Rb and 10 Torr of Neon buffer gas was used. The temperature of the cell was set to $\sim 65^\circ\text{C}$, providing a Rubidium vapor density of $\sim 4 \times 10^{11}/\text{cc}$. The cell was placed inside a four-layered magnetic shield, and a set of Helmholtz coils allowed us to determine the axial magnetic field. After the beams passed through the vapor cell they were separated using polarization optics. The pump beam was measured by a photo-diode detector and the probe beam was imaged onto a CCD camera. We used a small, $B_z = 50\text{ mG}$ axial magnetic field to set the quantization axis. A suitable frequency shift between the pump and the probe was introduced by scanning the EIT resonance and setting the pump AOM frequency to its center.

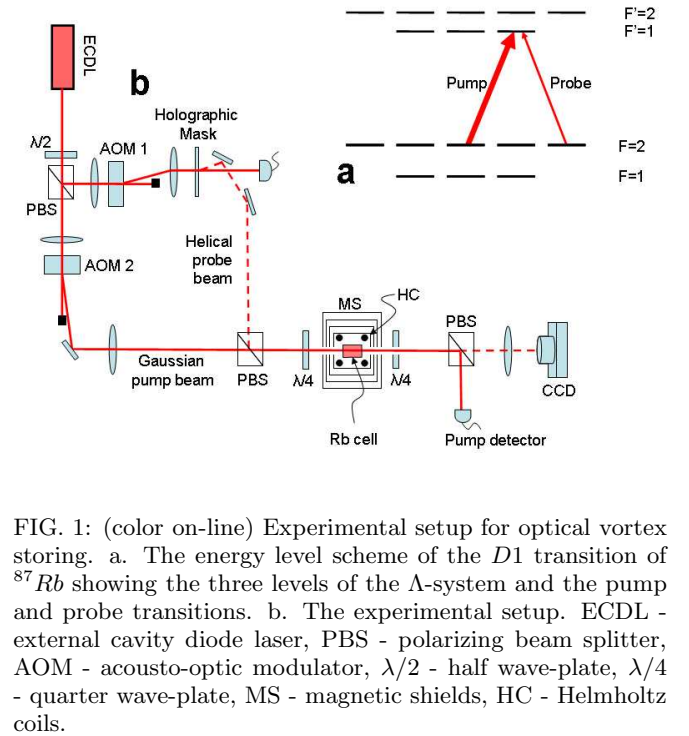


FIG. 1: (color on-line) Experimental setup for optical vortex storing. a. The energy level scheme of the $D1$ transition of ^{87}Rb showing the three levels of the Λ -system and the pump and probe transitions. b. The experimental setup. ECDL - external cavity diode laser, PBS - polarizing beam splitter, AOM - acousto-optic modulator, $\lambda/2$ - half wave-plate, $\lambda/4$ - quarter wave-plate, MS - magnetic shields, HC - Helmholtz coils.

A typical experimental sequence started by applying the pump beam for a long duration, optically pumping a substantial atomic population to the $|F = 2, m_F = +2\rangle$ state. A Gaussian probe pulse with $\sigma = 30\text{ }\mu\text{s}$ was then sent into the cell, and was slowed to a group velocity of $\sim 1000\text{ m/s}$ (delay of $\sim 50\text{ }\mu\text{s}$). During the passage of the probe pulse in the vapor cell, the pump beam was

turned off, storing the probe beam in the atomic ground-state coherence. After a certain storage duration (during which diffusion of the atoms occurred) the pump beam was turned back on, restoring the probe, which was then imaged onto a CCD camera. The CCD camera was triggered to measure *only* the restored part of the probe (to reduce the noise, we typically averaged 128 single shot images). The effect of atomic diffusion on the spatial shape of the restored light beam was studied by measuring it for different storage durations.

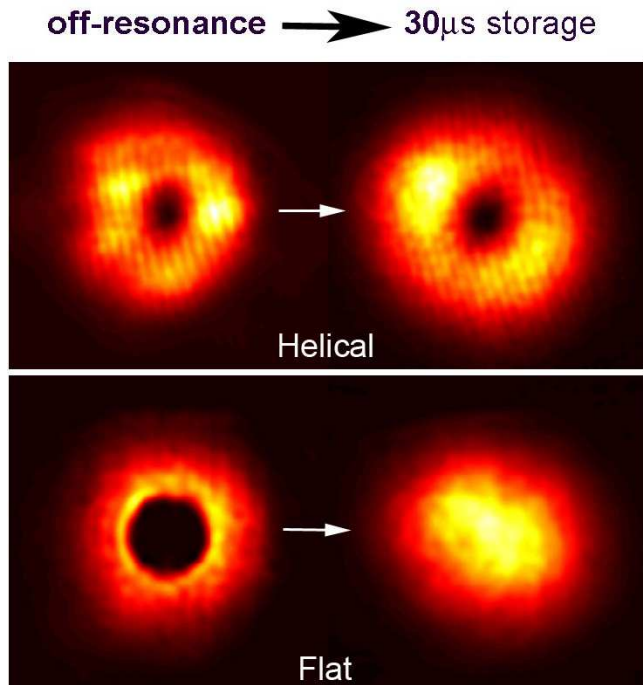


FIG. 2: (color on-line) The effect of diffusion on a helical beam and a flat phase beam. The left column shows the initial shape of a helical (top) and a flat phase (bottom) beams before any diffusion (off-resonance). The right column shows both beams after storage of $30 \mu s$. The center remains dark in the helical beam, while completely filling with light in the flat phase beam.

In the main experiment we studied the effect of diffusion on the spatial shape of a helical beam with $m = 1$. In a control experiment we used a transverse Gaussian beam (TEM_{00} mode, $w_0 = 670 \mu m$) with a darkened center that was created by using a small circular beam-stop, and which we imaged into the cell center. We also verified that the dark center of the beam persists throughout the length of the cell, by first measuring it along a parallel optical path of equal length outside the magnetic shield. The radius of the circular stop was $r_0 = 300 \mu m$. While the intensity pattern (lower-left image in Fig. 2) is also ring-shaped and similar to that of helical beam (upper-left image in Fig. 2), the partially-blocked Gaussian beam had a flat phase, as opposed to the twisted phase of the helical beam. Note that the actual radius

of the dark center of the flat phase beam when imaged on the atoms was $\sim 400 \mu m$, since we used an imaging system with a magnification of $M = 1.4$.

The main result of this work is depicted in Fig. 2. The left column shows the two beam-shapes we tested before any interaction with the atoms (the images were taken by detuning the laser from the atomic resonance). The right column shows the same beams after $30 \mu s$ of storage in the atomic vapor, when substantial diffusion took place. It is evident that while the central hole in the flat phase beam is completely filled, the central hole of the helical beam remains dark.

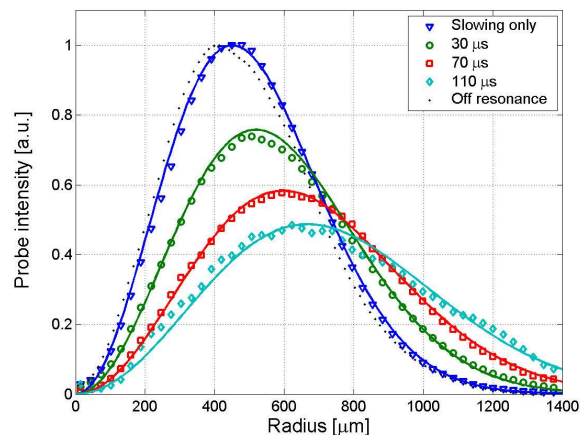


FIG. 3: (color on-line) Averaged radial cross-sections of the restored helical beam for different storage durations ($30 \mu s$, $70 \mu s$ and $110 \mu s$). The cross-sections of the off-resonance and slowing cases are also shown (the small difference between them is mainly due to diffusion during the slowing). It is evident that the center remains completely dark, even for the longest storage duration. The theoretical prediction (solid lines) are in excellent agreement to the experimental results (symbols). As explained in the text, the effect of diffusion on this transverse mode is to increase its waist w_0 to $\sqrt{w_0^2 + 4Dt}$, where $D = 11 \text{ cm}^2/s$.

In figures 3 and 4 we present a quantitative comparison between the calculated (solid lines) and the measured (symbols) cross-sections of the retrieved helical and flat phase beams, respectively, for different storage times. The total intensity of the stored beams decayed exponentially with a measured decay rate of $20,000 \text{ s}^{-1}$. To facilitate the comparison between the cross-sections, both the measured and the calculated curves are normalized to have same total intensity.

Since diffusion was strong in this experiment, we found that even without storing, the effect of the diffusion on the slowed beam shape was already pronounced. In fact, the dark center of the flat phase beam came out partly filled when slowed (the slowing delay time was $\sim 50 \mu s$). To account for this effect, the predicted curves of the retrieved cross-sections at different storage durations,

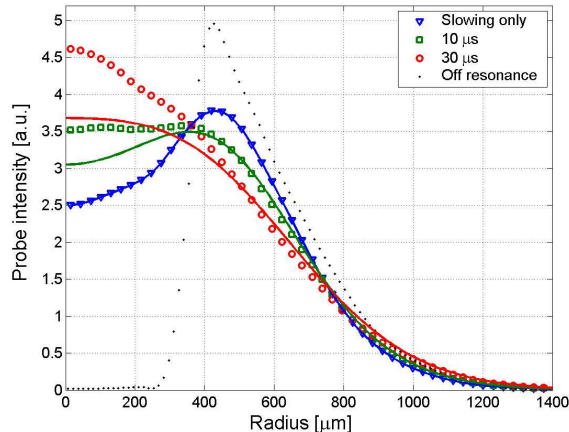


FIG. 4: Averaged radial cross-sections of the restored flat phase beam with a dark center for two storage durations (10 μs , 30 μs). Also shown are the slowed and off-resonance cross-sections. Due to the diffusion the dark center is already filled when the beam is only slowed. The theoretical prediction (solid lines) capture this essential phenomenon, but is less accurate than for the helical case. After storage time of 30 μs the dark center was completely filled with light.

shown in Fig. 3 and Fig. 4, were calculated by taking slowed beam profiles (helical or flat phase respectively) and propagating them forward in time, by the proper storage duration, using the three-dimensional diffusion propagator. Alternatively, we found that we can obtain the shape of the slowed beam by propagating the off-resonance beam (helical or flat phase) by an effective storage duration using Eq. 1 (for the helical beam) or Eq. 2 (for the flat phase beam). By further propagating in time by the actual duration of the storage, we were able to recover the predicted radial cross-section after storage, also in reasonable agreement with the measured data.

The best fit to the experiment was achieved when the diffusion coefficient was $D = 11 \text{ cm}^2/\text{s}$. This is in good agreement with an independent measurement of the diffusion coefficient we performed using optical pumping, and also in reasonable agreement with [18]. From Fig. 3 and Fig. 4 it is apparent that the theoretical prediction are able to capture the effect of the diffusion on the retrieved signal. More importantly, it is apparent that the dark center of the stored helical beam stays dark for times up to 110 μs , in clear contrast to the flat phase beam.

In summary, we have shown that it is possible to store and retrieve an optical vortex. We found that due to its topological nature, the dark center is stable to diffusion of the atoms. Since diffusion is homogeneous and isotropic, atoms enter the dark center from all directions

carrying a phase which is uniformly distributed over the unit circle. Upon reconstruction, they destructively interfere thus maintaining the darkness. We compared this situation to the case where the stored coherence is flat phased and with a dark center. We found that in this case the dark center is illuminated upon retrieval, already after 10 μs , showing that atoms indeed diffuse into the dark center. Propagating the initial stored coherence field with the diffusion propagator, we were able to predict the shape of the retrieved light and found good agreement with the measured data.

These results indicate a few interesting directions that we wish to point out. Since the position of the dark center is sensitive to phase gradients [19], one can make a grid of helical beams, and measure the transverse magnetic field gradient by comparing the stored image to the original one. Also, by using the dark spot as a pixel, different two dimensional patterns can be stored. Another possibility is to store more elaborate types of topological structures such as knots [20] or skyrmions [21]. Although the latter requires a double EIT scheme, which is experimentally more challenging, it holds the prospect of introducing non-Abelian phases that might be used to store quantum information in a novel way [22].

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